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Modelling of the homogeneous turbulence dynamics of stably stratified media

B. A. KOLOVANDIN and V. U. BONDARCHUK
A. V. Luikov Heat and Mass Transfer Institute, 220072, Minsk, Belarus

and

C. MEOLA and G. DE FELICE,
Facolta di Engegneria dell Universita di Napoli, Napoli, Italy

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Abstract—Numerical simulation of the dynamics of homogeneous turbulence of a stably stratified fluid in the presence of a vertical constant-density gradient was carried out. The second-order model which is universal with respect to turbulent Reynolds and Peclet numbers and molecular Prandtl number is applied to the numerical simulation of the dynamics of the velocity and density field parameters up to the final stage of decay. At small evolution times $N\tau$, the results of similation are compared with the familiar experimental data. The structure of a 'relict' turbulence was investigated for the molecular Prandtl numbers corresponding to air and sea water.

1. INTRODUCTION

A KNOWLEDGE of the laws governing the evolution of the homogeneous turbulence in a stably stratified liquid is of great theoretical and practical interest. Theoretically, this interest is motivated by the possibility of a more detailed (than in the case of the general form of shear turbulence) study of the role of density gradient as a turbulence generator in an actively stratified medium. From the practical point of view, the homogeneous turbulence of a stably stratified fluid is quite a realistic model of the upper atmosphere or of the thermohalocline in the ocean.

Over the past years considerable experimental efforts have been devoted to the elucidation of the fundamental aspects of the evolution of stably stratified homogeneous turbulence for both liquid (salt solution) and air. In the paper by Dickey and Mellor [1] which seems to be the first statistical–parametric experimental investigation of turbulence in a stably stratified fluid, a wavy change in the dispersion of vertical velocity fluctuations has been detected on the attainment of a certain dimensionless time $N\tau$ (here $N = ((g/\bar{\rho})\, \mathrm{d}\bar{\rho}/\mathrm{d}z)^{1/2}$ is the Brunt–Väisälä frequency) which presumably is indicative of the transition of turbulent fluctuations into internal waves. Moreover, at this very value of $N\tau$, a jumpwise decrease in the rate of kinetic energy dissipation of disturbances ε_u was detected confirming the presence of transition to a weakly dissipative wave field (in subsequent experiments such a behaviour of ε_u was not detected). The wave behaviour of the energy of lateral velocity fluctuations has been confirmed for the first time by Riley *et al.* [2] who numerically investigated the evolution of homogeneous turbulence in a stably stratified medium. However, in this case no jumpwise change in the parameter ε_u has been detected which is indicative of the absence of the dominating contribution of internal waves into the perturbed velocity field, at least, for the range of $N\tau$ values studied.

In more detailed (than in ref. [1]) experimental works of Stillinger *et al.* [3] and Itsweire *et al.* [4], a great number of 'energy' properties of velocity and density fields, different characteristic length scales, relating to velocity and density fields and also the transverse turbulent density flux were studied. The latter parameter has turned out to be extremely important for interpreting the specific features associated with the role of gravitation in the evolution of homogeneous turbulence. It has been shown that the considered parameter falls down to zero (the so-called collapse of turbulence), then, as $N\tau$ increases, it acquires negative values, i.e. the counter-gradient density transfer is observed. When in the earlier studies it was assumed that the condition $\overline{u_2\rho} = 0$ signifies a complete suppression of vertical velocity fluctuation and transition of disturbed field to the so-called two-dimensional 'fossil' turbulence, it has been demonstrated in refs. [3, 4] that gravitation slows down (as compared with the case of passive stratification) the decay of vertical fluctuations, i.e. downstream at the 'collapse' point they remain even more substantial than in the absence of the effect of gravitation. It was shown in those works that after the point where $\overline{u_2\rho} = 0$, a wavy change in all the measured parameters was observed; moreover a 'discontinuity' in the law of decay of the energy of vertical fluctuations was noted: the values of $\overline{u_2^2}$, averaged over the amplitude of 'waves', degenerate self-similarly after the collapse point, just as before it, but with a smaller

NOMENCLATURE

a_u	absolute constant, equation (2.1)
a_{ut}	variable coefficient, equation (2.3)
b_{ij}	deviator of P_{ij} tensor
b_u	variable coefficient, equation (2.1)
c_{ij}	deviator of D_{ij} tensor
D_{ij}	second-rank tensor, equation (2.1)
d_u	parameter of turbulent Reynolds number
Fr	Froude number, NM/U
F_u	'interaction function', equation (2.4)
g_i	gravity acceleration vector
K	kinetic energy of vertical fluctuations, $(1/2)\bar{\rho}_0\overline{u_2^2}$
L_b	buoyancy scale, $\overline{u_2^2}/N^2$
L_t	'overturning' scale, $\overline{\rho^2}^{1/2}/(\mathrm{d}\bar{\rho}_0/\mathrm{d}x_2)$
M	size of grid cell
N	Brunt–Väisälä frequency, $((\bar{g}/\rho_0)\,\mathrm{d}\bar{\rho}/\mathrm{d}z)^{1/2}$
n_{ts}	variable coefficient, equation (2.3)
P	potential energy of turbulence, $(1/2)\bar{\rho}_0(g/\bar{\rho}_0)\overline{\rho^2}/(\mathrm{d}\bar{\rho}/\mathrm{d}x_2)$
p	pressure fluctuations
P_{ij}	second-rank tensor of Reynolds stresses production, equation (2.1)
P_λ	turbulence Peclet number, $\overline{q^2}^{1/2}\lambda_\rho/\kappa$
R	time scale ratio parameter, τ_u/τ_ρ
R_λ	turbulence Reynolds number, $\overline{q^2}^{1/2}\lambda_u/\nu$
S_{ij}	second-rank tensor of mean shear
T_{BV}	Brunt–Väisälä period, $2\pi/N$
t	temperature fluctuations
U_i	mean velocity vector.

Greek symbols

α_u	variable coefficient, equation (2.1)
β_u	variable coefficient, equation (2.1)
γ_u	variable coefficient, equation (2.1)
δ_u	coefficient in parameter d_u
Δ	Laplace operator
ε_u	rate of kinetic energy turbulence dissipation
ε_t	rate of temperature fluctuation degradation
κ	molecular diffusion
λ_t	Taylor microscale of scale field
λ_u	Taylor microscale of velocity field
ν	kinematic viscosity
ξ	vector of distance between two points
ρ	density fluctuations
$\bar{\rho}$	mean density
σ	molecular Prandtl number, ν/κ
τ	time
τ_u	time scale of velocity field, $\overline{q^2}/\varepsilon_u$
τ_ρ	time scale of density field, $\overline{\rho^2}/\varepsilon_\rho$.

Subscripts

a	asymptotic value
s	condition of strong turbulence
T	relates parameter to turbulized medium
t	relates function to temperature field
u	relates function to velocity field
w	condition of weak turbulence
0	absence of mean shear.

exponent in the power law (of course, all the above relates to relatively large turbulent Reynolds and Peclet numbers that correspond to the experiments mentioned).

In a more comprehensive experimental work of Lienhard and Van Atta [5], carried out for a temperature-stratified air, the concept of homogeneous turbulence of a stably stratified medium has been developed as an essentially two-scale process in which large-scale perturbations are controlled by buoyancy forces and the fine-scale structure by viscosity. In the light of this concept it has been shown that the condition $\overline{u_2\rho} = 0$ does not mean the disappearance of the active turbulent mixing but is the manifestation of an important trend in the stratified flow turbulence which was discovered in the work, i.e. that at a certain value of the 'phase' $N\tau$ the vertical mass fluxes, associated with large and small vortices, are equal in magnitude and opposite in sign. The change in the sign of the mass flux (counter-gradient transfer) is associated exclusively with large-scale turbulent motions and results from restratification, i.e. the consequence of the motion of large vortices which were 'thrown' by turbulence from the region of high to the region of small density, under the action of buoyancy to the equilibrium and then to the region of higher density. As to the mass flux associated with fine vortices, it is down-gradient at any time instant.

Thus, the scales at which turbulent mixing takes place and the scales on which the generation of internal waves occurs and which manifests itself in the alternation of the mass flux sign, are greatly spaced in the spectrum of the scales. As to the mechanism of energy transfer from relatively fine-scale turbulence to large-scale wave motion, then it is not as yet entirely clear. In order to elucidate this extremely important problem, a number of attempts at direct numerical simulation of the evolution of homogeneous turbulence of a stratified fluid have been undertaken. The first effort has been made by Riley *et al.* [2]. The authors of that work have shown that even though stratification increases the decay rate of the dissipation of the kinetic energy disturbances, this parameter remains rather high for the turbulence energy to be regarded as fully converted into the energy of internal waves. Thus, it follows from that work that at least in the studied range of the dimensionless time $N\tau$, the internal gravitation waves coexist with the turbulence proper as with a purely stochastic field.

A detailed numerical investigation of the stochastic and wave modes in the turbulence of a stratified medium was carried out in a recent work of Metais and Herring [6].

The results obtained in the above-mentioned numerical studies show that the development of a homogeneous turbulence in a stably stratified medium have been studied at least for moderate turbulent Reynolds and Peclet numbers. In practical situations existing in the atmosphere and ocean, the conditions of small values of R_λ and P_λ numbers (atmosphere) and small R_λ and moderate or even large P_λ (the thermohalocline in the ocean) are being realized. Therefore, when the above-mentioned numerical works have some relevance to the atmosphere, they practically do not have direct bearing on the ocean, since the case $P_\lambda \gg R_\lambda$ is as yet inaccessible for direct numerical simulation. Moreover, great evolution times at which $R_\lambda \to 0$ are also inaccessible as yet for direct numerical simulation.

In the present work an attempt has been made to apply the second-order model, developed by one of the authors [7], to the study of the dynamics of the stably stratified homogeneous turbulence in a Boussinesq fluid. In so doing the following problems were posed:

- the study of the dynamics of relatively strong turbulence at different values of the general Froude number, $Fr = NM/U$;
- the study of the effect of molecular Prandtl number on the evolution (decay) of moderately strong stratified turbulence;
- the study of the transition of moderately strong stratified turbulence to a weak relict turbulence and the elucidation of the relative role of internal waves and of the turbulence proper at a very large time of evolution.

2. GOVERNING EQUATIONS, SCALES AND PARAMETERS

The second-order model of homogeneous turbulence of a stably stratified Boussinesq fluid consists of the following differential equations (the details relating to the allowance for gravitation in different model equations are omitted).

2.1. *Model equation for the tensor of Reynolds stresses*
This is given by

$$\frac{\mathrm{D}}{\mathrm{D}\tau}\overline{u_iu_j} - P_{ij} + 2\varepsilon^{ij} - \Phi_{ij} = -\beta(g_i\overline{u_jt} + g_j\overline{u_it}),$$

where

$$P_{ij} = -(\overline{u_iu_k}\,\mathrm{d}U_j/\mathrm{d}x_k + \overline{u_ju_k}\,\mathrm{d}U_i/\mathrm{d}x_k)$$

is the second-rank tensor of Reynolds stress production at the expense of the mean velocity shear;

$$\varepsilon^{ij} \equiv \nu(-\Delta_\xi\overline{u_iu_j'})_{\xi=0} = \tfrac{1}{3}(d_ua_{ij} + \delta_{ij})\varepsilon_u$$

is the model relation for the rate of dissipation of Reynolds stresses; $\varepsilon_u = \varepsilon^{ii}$ is the dissipation of turbulence kinetic energy; $a_{ij} = 3\overline{u_iu_j}/\overline{q^2} - \delta_{ij}$ is the deviator of the $\overline{u_iu_j}$ tensor;

$$\Phi_{ij} = \overline{\frac{1}{\rho}p\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)} = \Phi_{ij}^{(1)} + \Phi_{ij}^{(2)} + \Phi_{ij}^{(3)}$$
$$= -\,a_u(1-d_u)a_{ij}\varepsilon_u - b_u[\alpha_u\overline{q^2}S_{ij} + (\beta_ub_{ij} + \gamma_uc_{ij})P_{kk}]$$
$$-\tfrac{1}{10}\beta g_k[2\overline{u_kt}\delta_{ij} - 3(\overline{u_it}\delta_{jk} + \overline{u_jt}\delta_{ik})]$$

is the model relation for the second-rank tensor of the interaction of pressure fluctuations with the gradients of velocity fluctuations the three constitutive parts of which model, respectively, the 'slow' return to the isotropy, 'rapid' deformation of turbulence by mean shear and contribution of gravitation; $\overline{q^2} = \overline{u_i^2}$ is the doubled kinetic energy of turbulence; $S_{ij} = \frac{1}{2}(\mathrm{d}U_i/\partial x_j + \mathrm{d}U_j/\partial x_i)$ is the second-rank tensor of mean shear; $b_{ij} = 3P_{ij}/P_{kk} - \delta_{ij}$ is the deviator of the P_{ij} tensor; $c_{ij} = 3D_{ij}/P_{kk} - \delta_{ij}$ is the deviator of the D_{ij} tensor; $D_{ij} = -(\overline{u_iu_k}\ \mathrm{d}U_k/\mathrm{d}x_j + \overline{u_iu_k}\ \mathrm{d}U_k/\mathrm{d}x_i)$; $\beta = -(1/\bar{\rho}_0)(\mathrm{d}\bar{\rho}/\mathrm{d}T)_0$ is the thermal expansion coefficient of the medium; a_u, b_u, d_u, α_u, β_u, and γ_u are the coefficients of the model which generally are some functions of governing parameters. For the homogeneous turbulence considered these parameters are: the turbulent Reynolds number $R_\lambda = \overline{q^2}^{1/2}\lambda_u/\nu$; the ratio of the turbulent kinetic energy production rate to the rate of its dissipation $\bar{P}_u = P_{kk}/2\varepsilon_u$; the Taylor microscale of the length λ_u which in the considered homogeneous anisotropic turbulence is determined by the relation

$$\lambda_u^2 = 5\nu\overline{q^2}/\varepsilon_u = 5\nu\tau_u,$$

where $\tau_u = \overline{q^2}/\varepsilon_u$ is the time scale relating to the turbulent velocity field.

2.2. *An exact equation for the mean square of scalar fluctuations*

$$\frac{\mathrm{D}}{\mathrm{D}\tau}\overline{t^2} + 2(1-\bar{P}_t)\varepsilon_t = 0,$$

where $\varepsilon_t = \kappa\overline{(\partial t/\partial x_k)^2} \equiv \kappa(-\Delta_\xi\overline{tt'})_{\xi=0}$ is the rate of 'smearing' of scalar fluctuations; $\bar{P}_t = P_{tt}/2\varepsilon_t$ is the ratio of the rate of scalar fluctuation production due to the vector of the gradient of its mean value $P_{tt} = -2\overline{u_kt}\,\partial T/\partial x_k$ to the rate of destruction ε_t.

2.3. *The model equation for the scalar quantity flux vector*
This is given by

$$\frac{\mathrm{D}}{\mathrm{D}\tau}\overline{u_it} - P_{it} + \varepsilon_{it} - \Phi_{it} = -\beta g_i\overline{t^2},$$

where

$$P_{it} = -(\overline{u_i u_k}\,\mathrm{d}T/\mathrm{d}x_k + \overline{u_k t}\,\mathrm{d}U_i/\mathrm{d}x_k)$$

is the vector of turbulent flux production by the gradients of the mean values of the velocity and scalar quantity, $\varepsilon_{it} = d_u 1/\tau_{utw}\overline{u_i t}$ is the model relationship for the rate of flux $\overline{u_i t}$ 'smearing'

$$\Phi_{it} \equiv \frac{1}{\rho}\overline{p\frac{\partial t}{\partial x_i}} = \Phi_{it}^{(1)} + \Phi_{it}^{(2)} + \Phi_{it}^{(3)}$$

$$= -a_{ut}(1-d_u)\frac{1}{\tau_{uts}}\overline{u_i t} + \tfrac{1}{5}(4\mathrm{d}U_i/\mathrm{d}x_k - \mathrm{d}U_k/\mathrm{d}x_i)\overline{u_i t} + \tfrac{1}{3}\beta g_k(\mathrm{d}_u a_{ik} + \delta_{ik})\overline{t^2}$$

is the model relation for the vector of pressure with the gradients of a scalar interaction (the relations for $\Phi_{it}^{(2)}$ and $\Phi_{it}^{(3)}$ are exact),

$$a_{ut}\tau_{uts}^{-1} = [4 - F_{us} + n_{ts}^{-1}(\overline{u_n^2}/\overline{q^2})R - \bar{P}_u]\tau_u^{-1}$$

is the model relationship for the mixed time scale τ_{ut}, for $R_\lambda \gg 1$, $P_\lambda \gg 1$, $\bar{P}_u = \text{const.}$, $\bar{P}_t = \text{const.}$;

$$\tau_{utw}^{-1} = [2(\sigma_{T\alpha w 0} + 3/5)R/R_{\alpha w 0} - \bar{P}_u]\tau_u^{-1}$$

is the model relationship for the mixed time scale for $R_\lambda \ll 1$, $P_\lambda \ll 1$, $\bar{P}_u = \text{const.}$, $\bar{P}_t = \text{const.}$;

$$\sigma_{T\alpha 0} = (3/10)\frac{1-\sigma}{\sigma}\Bigg/\left[1 - \left(\frac{2\sigma}{1+\sigma}\right)^{3/2}\right]$$

is the asymptotic (for a large evolution time) turbulent Prandtl number for $R_\lambda \ll 1$, $P_\lambda \ll 1$, $\bar{P}_u = \text{const.}$, $\bar{P}_u \to 0$ (Deissler's relation [8]); $P_\lambda = \overline{q^2}^{1/2}\lambda_t/\kappa$ is the turbulent Peclet number, $\lambda_t^2 = 6\kappa\overline{t^2}/\varepsilon_t = 6\kappa\tau_t$ is the squared Taylor microscale of length relating to a scalar field; $\tau_t = \overline{t^2}/\varepsilon_t$ is the time scale relating to a scalar field;

$$R_{\alpha 0} = \frac{1}{5}\left[1 - 2\left(\frac{2\sigma}{1+\sigma}\right)^{3/2} + \sigma^{3/2}\right]\Bigg/\left[1 - \left(\frac{2\sigma}{1+\sigma}\right)^{1/2} + \sigma^{1/2}\right]$$

is the parameter of the time scales τ_u/τ_t ratio of homogeneous turbulence for $R_\lambda \ll 1$, $P_\lambda \ll 1$. $\bar{P}_u = 0$, $\bar{P}_t = \text{const.}$ (exact solution of Dunn and Reid [9]).

2.4. *A model equation for the rate of turbulent kinetic energy dissipation*

This is given by

$$\frac{\mathrm{D}}{\mathrm{D}\tau}\varepsilon_u + (F_u^{**} - 3\bar{P}_u)\varepsilon_u^2/\overline{q^2} = -2d_u\beta g_i\frac{\sigma}{1+\sigma}\frac{1}{\tau_{utw}}\overline{u_i t},$$

where

$$(F_u^{**} - 3\bar{P}_u)\varepsilon_u^2/\overline{q^2} = GEN + DIS + IN$$

is the model relation for the sum of three effects: production of ε_u by mean shear ($GEN = 0$ for $R_\lambda \gg 1$), destruction of the parameter ε_u (or of vorticity $\overline{\omega_u^2} = (1/\nu)\varepsilon_u$) and change in $\overline{\omega_u^2}$ by means of stretching (compression) of vortex filaments (see for details ref. [7]).

The right-hand side of the equation considered simulates the effect of gravitation on the vorticity of velocity fluctuation field.

The coefficient F_u^{**}, associated with the total effect of vorticity destruction and stretching of isotropic velocity field, is modelled by the turbulent Reynolds number function

$$F_u^{**}(R_\lambda) = F_{us}^{**}(1-d_u) + F_{uw}^{**}d_u$$

where $F_{us}^{**} = 11/3$ and $F_{uw}^{**} = 14/5$ are the asymptotic values of F_u^{**} for $R_\lambda \gg 1$ and $R_\lambda \ll 1$ which are found analytically by the known procedure.

The parameter d_u is the empiric function of the turbulent Reynolds number. In the present study it is specified in the form

$$d_u = 1 - 2(1 + \sqrt{(1 + \delta_u/R_\lambda^2)})^{-1}.$$

The remaining coefficients of the model have the form (for details see ref. [7]):

$$b_u = 2\{1 + [(\alpha_s/F_{us}^{**} - 2)(1-d_u) + (\alpha_w/F_{uw}^{**} - 2)d_u]\bar{P}_u\}^{-1},$$

$$\alpha_u = 2(\tfrac{1}{11}d_u - \tfrac{1}{10}),\quad \beta_u = \tfrac{4}{33}d_u,\quad \gamma_u = -\tfrac{1}{33}d_u,$$

$$\alpha_u \simeq 3,\quad \delta_u \simeq 2800,\quad \alpha_s \simeq 1,\quad \alpha_w \simeq 0.5.$$

2.5. *A model equation for the rate of 'smearing' of scalar quantity fluctuations*

$$\frac{\mathrm{D}}{\mathrm{D}\tau}\varepsilon_t = 2\mathrm{d}\bar{P}_u\varepsilon_t\frac{1}{\tau_u} + 2\mathrm{d}\bar{P}_t\frac{1}{1+\sigma}\varepsilon_t\frac{1}{\tau_{utw}} - (F_{t1}^* + F_{t2}^*R)\varepsilon_t\frac{1}{\tau_u}$$

where the first two terms on the right-hand side of the equation model the effect of ε_t production by mean velocity shear and by the gradient of mean value of the scalar quantity (in strong turbulence this effect is absent); the second two terms simulate the joint effect of the production of ε_t by deformation of vortex filaments of velocity field and associated scalar field 'formations' and also the destruction of parameter ε_t by molecular diffusion.

The coefficients F_{t1}^* and F_{t2}^* are modelled by the following functions of turbulent Reynolds and Peclet numbers and also of the parameters P_u and P_t:

$$F_{t1}^* = F_{t1}^{**} - \bar{P}_u + \mathrm{d}f_1\bar{P}_u,$$

$$F_{t2}^* = F_{t2}^{**} - 2\bar{P}_t + \mathrm{d}\left(\frac{2\bar{P}_t}{1+\sigma}\right)f_2,$$

$$f_1 = \left(1 - \frac{2\bar{P}_t}{1+\sigma}\right) + 1,$$

$$f_2 = \left[2(\sigma_{T\alpha 0} + \tfrac{3}{5}) - \tfrac{4}{3}(R_{\alpha 0} - \tfrac{3}{5})\left(\frac{2\bar{P}_t}{1+\sigma}\right)^{-1}\right]R_\alpha^{-1},$$

$$f_3 = \tfrac{4}{3}(R_{\alpha 0} - \tfrac{3}{5})\frac{R}{R_{\alpha 0}}\left(1 - \frac{2\bar{P}_t}{1+\sigma}\right)\left(\frac{2\bar{P}_t}{1+\sigma}\right)^{-1}.$$

The coefficients F_{t1}^{**} and F_{t2}^{**} model the above-indicated effects in the case of the velocity and scalar field isotropy in the form of the following functions of the turbulent Reynolds number

$$F_{t1}^{**} = F_{u}^{**} - 2 - \tfrac{4}{3}d,$$

$$F_{t2}^{**} = 2 + \tfrac{4}{3}d.$$

3. DYNAMICS OF TURBULENCE IN A STABLY STRATIFIED FLUID

Based on the considered second-order model, the present section investigates the dynamics of homogeneous turbulence of a stably stratified fluid in the absence of mean velocity shear. It is assumed that in an infinite fluid volume with a constant vertical mean density gradient $\mathrm{d}\bar{\rho}/\mathrm{d}x_2 = \text{const.}$ (the coordinate x_2 is directed vertically, opposite to the gravity acceleration) the evolution of turbulence is going on from the initial state corresponding to a three-dimensional homogeneous isotropic turbulence.

As is known, in a stably stratified fluid disturbed from equilibrium fluctuations originate internal gravitational waves with the Brunt–Viäsälä frequency

$$N = \left(-\frac{g}{\rho_0}\frac{\mathrm{d}\bar{\rho}}{\mathrm{d}x_2}\right)^{1/2}.$$

In the case of a turbulized fluid, the elements of the fluid with a wide spectrum of turbulent fluctuations become involved into disturbances of gravitational nature. The turbulent modes which have the frequencies commensurable with the Brunt–Viäsälä frequency turn out to be affected by the buoyancy forces. In the vertical direction the displacement of the turbulized element is estimated by the so-called overturning scale

$$L_t = \overline{\rho^2}^{1/2}/(\mathrm{d}\bar{\rho}_0/\mathrm{d}x_2),$$

where $\overline{\rho^2}$ are the mean-squared density fluctuations. This length scale characterises the size of large turbulent modes in the density stratified fluid.

The 'adjustment' of the turbulence, developing in time, to the internal gravitational waves can be easily illustrated on the example of grid-generated turbulence. At small evolution times when turbulence is still virtually isotropic, the well-known relations for the turbulence energy and Taylor length scale are fulfilled

$$\overline{q^2} \sim \tau^{-1}, \quad \lambda_u \sim \tau^{1/2}.$$

It can be easily seen that the Froude number (here the quantity is used which is reciprocal of that generally used because of the fact that in the absence of gravitation the value of the parameter which determines the effect of gravitation is equal to zero).

$$Fr_T = N\lambda_u/\overline{q^2}^{1/2}$$

will evolve following the 'law'

$$Fr_T \sim N\tau.$$

This estimate shows that at small evolution times $Fr_T \ll 1$, i.e. gravitation does not influence turbulence. For the time $\tau \sim N^{-1}$ the parameter Fr_T attains the value of the order of unity, i.e. the effects of turbulence and buoyancy become commensurable. At these times of evolution, wavy-like changes of the statistical parameters of turbulence should begin.

Experimental data obtained in recent years for the turbulence of stably stratified fluid [3–5], which evolves in the wake behind a grid. show that wavy downstream variation of time-average parameters is one of the specific features of the evolution of stably stratified homogeneous turbulence. As a result of the indicated experiments a number of conclusions have been drawn about both the phenomenon of transition from three-dimensional turbulence to a presumably quasi-two-dimensional turbulence plus internal gravitational waves at relatively small distances from the grid and interaction between turbulence and internal waves after the collapse. However, the question of whether or not gravitation causes the transition of three-dimensional turbulence to quasi-two-dimensional or to a gravitational wave or to a superpositional field of turbulence and internal waves remains open up to now, since this can be elucidated only at high evolution times which as yet are inaccessible either for direct modelling of physical experiment.

In the present work an attempt has been made to apply an alternative approach to direct numerical simulation—to apply a second-order model which describes the dynamics of the coupled velocity and density fields for studying the transition of a three-dimensional turbulence in a stably stratified medium, to the final stage of evolution with a view to elucidate the structure of the field at this stage: whether it is a two-dimensional turbulence (as is predicted in some theoretical works) or this is the field of internal waves or the superposition of turbulent and wave modes.

In this situation, quite a reasonable question may arise: what is the basis for the belief about the adequate prediction on the basis of the proposed model of the behaviour of stably stratified turbulence at high evolution times? It can be answered with certainty that, first, because of the satisfactory agreement between the predicted and experimental results for the case of strong turbulence in a non-stratified fluid (see ref. [10]). Second, because the model matches exact asymptotics [9] with weak non-stratified turbulence, i.e. at very large evolution times and, finally, on the trivial account for the buoyancy effect in the differential equations of the model.

In what follows we shall try to interprete the numerical results obtained, to correlate them, when possible, with the experimental data of other authors; to investigate the degeneration of the stably stratified turbulence after the collapse having explained the role of the molecular Prandtl number; to investigate the character of the dynamics of stably stratified turbulence at small turbulence Reynolds and Peclet numbers, i.e. to elucidate whether or not it really tends to a two-dimensional state as it follows from a number of theoretical investigations.

3.1. *Dynamics of moderately strong turbulence of stably stratified fluid*

This section deals with the analysis of the results of numerical simulation for the dynamics of moderately strong turbulence in a stably stratified medium as implemented in the experiments of Itsweire *et al.* [4] for water and Lienbard and Van Atta [5] for air.

In particular, out of the multitude of experiments made for water, three have been selected that correspond to different values of the Froude number ($Fr = NM/U = 15.2\times10^{-2}$; 6.3×10^{-2}; 3.7×10^{-2}) with the grid cell measuring $M = 3.81\times10^{-2}$ m. Of the experiments with air [5], that one was selected which corresponded to the following parameters: $Fr = 2.4\times10^{-2}$, $M = 5.08\times10^{-2}$ m.

The numerical results given below were obtained as the solution of the Cauchy problem for a system of ordinary differential equations that describe the ensemble averaged turbulence parameters: $\overline{u_2^2}$, $\overline{q^2}$, ε_u, $\overline{\rho^2}$, ε_ρ, $\overline{u_2\rho}$; for air the latter three parameters are replaced by the quantities $\overline{t^2}$, ε_t and $\overline{u_2 t}$. The initial conditions were borrowed from the corresponding experiments.

The numerical results show that a turbulent flux of the scalar substance $\overline{u_2\rho}$ (Fig. 1) which determines the source gravitation terms in the equations for the parameters $\overline{u_2^2}$, $\overline{q^2}$ and ε_λ performs oscillations near the position where $\overline{u^2\rho} = 0$, with the first crossing of time axis at $N\tau \approx 2.5$; the oscillation period is assessed as

$$T \approx 3.5\,N^{-1},$$

irrespective of the Froude number value.

Oscillations of the function $\overline{u_2\rho}$ directly cause oscillations of lateral velocity fluctuations, $\overline{u_2^2}$ (see Fig. 2) and, via this parameter, of the kinetic energy of turbulence (Fig. 3). Less noticeable are the oscillations of the averaged value of horizontal velocity fluctuations, u_1^2 (Fig. 4) since gravitation effect for them manifests itself only through pressure fluctuations.

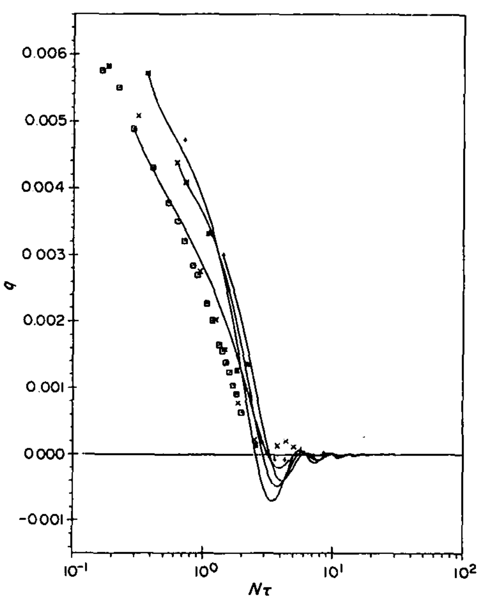


FIG. 1. Evolution of a transverse mass (or heat) flux. $q = \overline{u_2\rho}/UM(-\mathrm{d}\bar{\rho}/\mathrm{d}x_2)$. *, $Fr = NM/U = 0.037$; ×, $Fr = 0.063$; +, $Fr = 0.152$; –, experiment for water [4]; □, $Fr = 0.024$; —, experiment for air [5]; solid lines, numerical simulation.

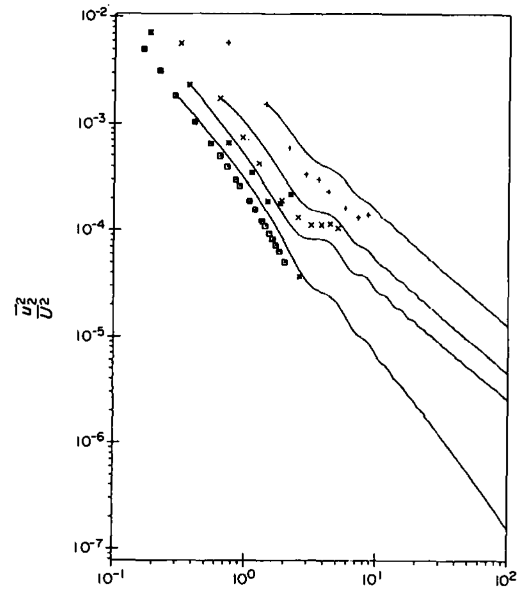


FIG. 2. Evolution of the kinetic energy of transverse velocity fluctuations, $\overline{u_2}/U^2$. For designations see Fig. 1.

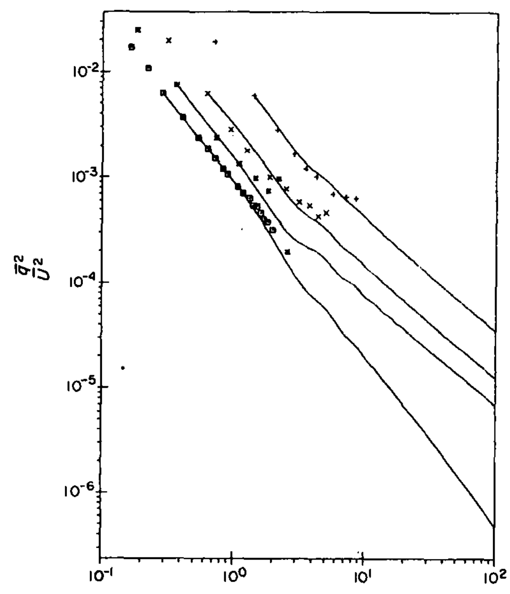


FIG. 3. Evolution of the full doubled kinetic energy of turbulence, $\overline{q^2}/U^2$. For designations see Fig. 1.

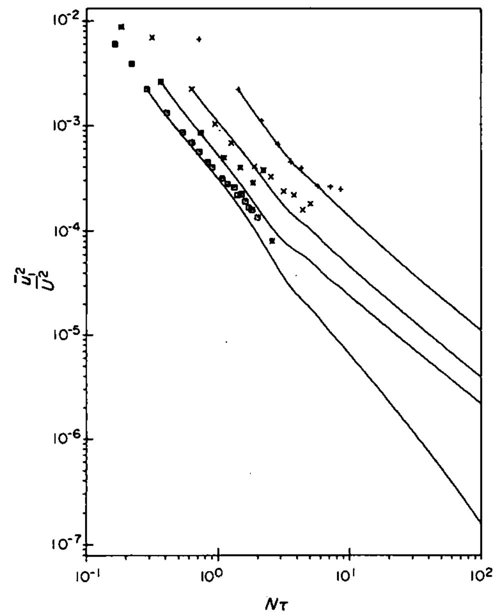


FIG. 4. Evolution of the kinetic energy of longitudinal velocity fluctuations, $\overline{u_1^2}/U^2$. For designations see Fig. 1.

Note that the absence of vertical turbulent mass (or heat) transfer occurring at $N\tau \simeq 30$ (see Fig. 5) does not at all mean the suppression of vertical velocity fluctuations, which, as follows from Fig. 2, decay with an averaged (over the amplitude oscillations) rate which does not exceed the rate of decay of the non-stratified fluid velocity fluctuations.

The oscillations of the parameter $\overline{u^2\rho}$ show up directly also on the behaviour of density fluctuations (Fig. 5) but not through the source gravitational term

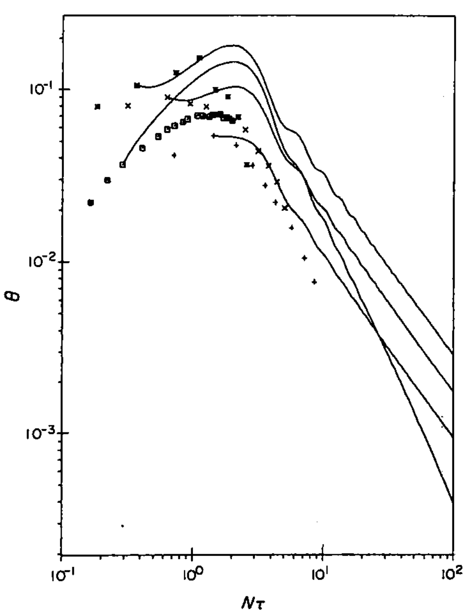


FIG. 5. Evolution of mean squared density (or temperature) fluctuations, $\theta = \overline{\rho^2}/M^2(\mathrm{d}\bar{\rho}/\mathrm{d}x_2)^2$. For designations see Fig. 1.

of non-gradient type (which is not naturally present in this equation) but through the rate of density fluctuation production by the mean density gradient

$$P_{\rho\rho} = -2\overline{u_2\rho}\,\mathrm{d}\bar{\rho}/\mathrm{d}x_2.$$

When $N\tau < 1$, the function $\overline{\rho^2}$ evolves in the fashion similar to the mean square of the passive scalar fluctuations (see, for example, ref. [10]). At $N\tau \simeq 2.5$, the decay of $\overline{\rho^2}$ begins which qualitatively is similar to the decay of the isotropic passive scalar.

A specific feature different from the isotropic case is the wavy-like oscillation in density fluctuations with a period equal to that of the transverse density flux oscillations.

As is known, the wavy-like oscillations of the turbulence parameters in a stably density-stratified medium indicates the presence of internal waves in the field excited by gravitational forces. As is known, the indicator of the transition of turbulence to internal gravitational waves is the value of the ratio of the kinetic energy of vertical velocity fluctuations

$$K = \tfrac{1}{2}\bar{\rho}_0\overline{u_2^2} = \tfrac{1}{2}\bar{\rho}_0 N^2(\overline{u_2^2}/N^2)$$

to the potential energy of the density field

$$P = \tfrac{1}{2}\bar{\rho}_0\frac{g}{\bar{\rho}_0}\frac{\overline{\rho^2}}{(\mathrm{d}\bar{\rho}/\mathrm{d}x_2)} = \tfrac{1}{2}\bar{\rho}_0 N^2\overline{\rho^2}/(\mathrm{d}\bar{\rho}/\mathrm{d}x_2)^2$$

or the ratio of characteristic length scales

$$\frac{K}{P} = \left(\frac{L_b}{L_t}\right)^2,$$

where $L_b = (\overline{u_2^2}/N^2)^{1/2}$ is the buoyancy scale, $L_t = \overline{\rho^2}^{1/2}/(\mathrm{d}\bar{\rho}/\mathrm{d}x)$ is the 'overturning' scale or the characteristic distance along the vertical over which the element of a turbulized fluid may displace from the equilibrium position. The upper limit of the parameter L_t is the scale L_b determined by the turbulence inertia.

It is obvious that at small evolution times of strong turbulence the value of the ratio (L_b/L_t) exceeds unity. In this case the contribution of the internal waves into the turbulent field is insignificant. The condition

$$\frac{K}{P} = 1$$

testifies to the 'parity' of turbulent velocity disturbances in the vertical direction and internal waves; sometimes, this condition is considered to be the condition for the transition of turbulence to internal waves, assuming that finally (at very large evolution time) turbulence will entirely pass over into the internal waves; in what follows it will be shown that such a transition is not always possible.

The curves presented in Fig. 6 show that in the region of a relatively strong turbulence (the turbulence Reynolds and Peclet numbers are presented in Figs. 7 and 8, respectively) the ratio L_b/L_t similarly to the transverse mass flux is an oscillating function attaining the asymptote. At small values of the dimensionless time, $L_b/L_t > 1$ indicating the prevailing con-

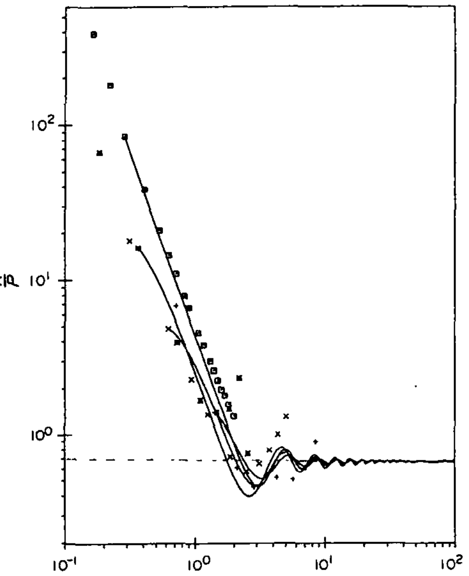


FIG. 6. Evolution of the ratio of the kinetic energy of vertical velocity fluctuations to the potential energy, $K/P = (\overline{u_2^2}/N^2)/(\overline{\rho^2}/(\mathrm{d}\bar{\rho}/\mathrm{d}x_2)^2 = (L_b/L_p)^2$. For designations see Fig. 1.

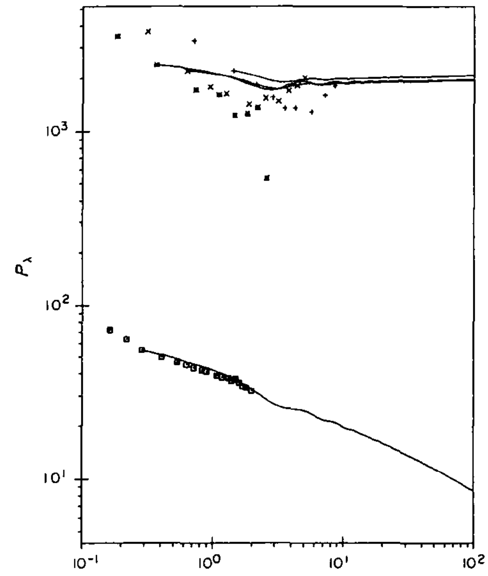


FIG. 8. Evolution of the turbulence Peclet number $P_\lambda = \lambda_\rho \overline{q^2}^{1/2}/\kappa$. For designations see Fig. 1.

tribution of turbulence into a perturbed field. The condition $L_b/L_t = 1$ corresponds to the coordinate $N\tau \simeq 2$.

The minimum value of the function K/P corresponds to the coordinate $N\tau \simeq 2.5$, i.e. to the point where the transverse mass flux vanishes for the first time. The asymptotic value which is attained by the ratio K/P is equal approximately to 0.7.

Since, as was shown above, the contribution of internal waves into the superposition field at $N\tau \simeq 2$ becomes equal to that of the turbulence proper, then starting from this point one should await the specific features of the dynamics of the considered parameters inherent in a stratified fluid. Indeed, from the data given in Fig. 7, it is seen that at $N\tau \simeq 2.5$ corresponding to the collapse of the transverse mass flux, a jumpwise decrease in the rate of turbulence inertia decay is observed. The reason for this seems to be internal waves with a large period (proportional to the Brunt–Väisälä period, $T_{TV} = 2\pi/N$); this is indicated by a jumpwise change in the increase in the Taylor macroscale of velocity field (Fig. 9) at $N\tau \simeq 2.5$.

Since the internal random waves are less dissipative structures than turbulence, then, starting from the same coordinate ($N\tau \simeq 2.5$), a slower decay of velocity fluctuations is observed (see Figs. 2–4).

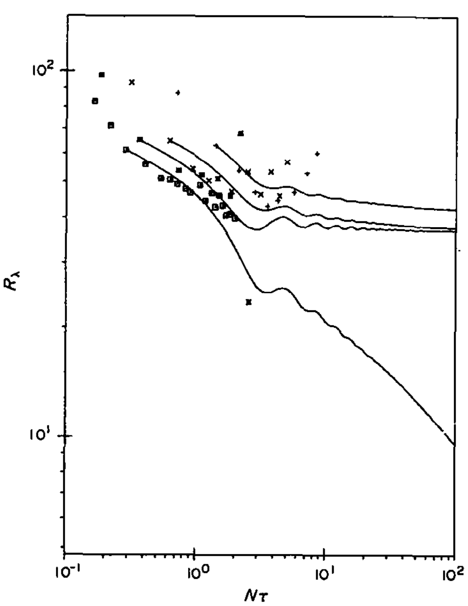


FIG. 7. Evolution of the turbulence Reynolds number, $R_\lambda = \lambda_u \overline{q^2}^{1/2}/\nu$. For designations see Fig. 1.

3.2. *Effect of molecular Prandtl number*

Realizations of the experiment considered in the present work and the results of simulation, corresponding to them, relate to a moderately strong turbulence of velocity field, as indicated by the pattern of turbulence Reynolds number evolution (Fig. 7). At the same time the scalar field turbulence is moderately strong for air and quite strong for water (see Fig. 8). In view of this, it can be expected that even at almost the same values of the global Froude number, and different values of molecular Prandtl number the dynamics of the turbulence parameters should be different. As follows from the above-mentioned figures, in the near region ($N\tau < 1$), where the effect of buoyancy forces on the velocity field is negligible, the

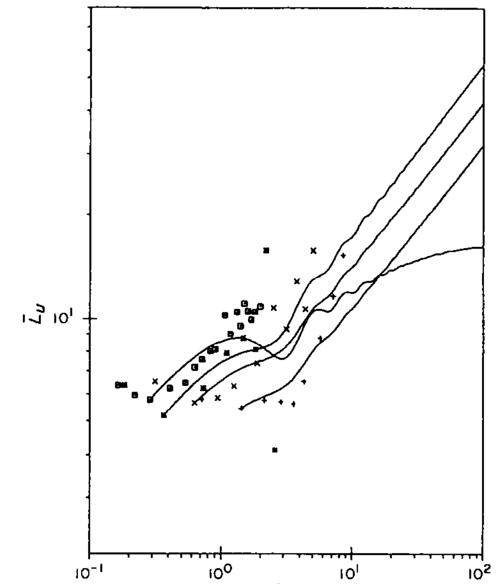


FIG. 9. Evolution of the Taylor macroscale of velocity fluctuations, $\bar{L}_u = L_u/M$. For designations see Fig. 1.

turbulence parameters behave similarly to the case of a passive scalar (see, for example, ref. [10]) : the velocity field parameters evolve self-similarly and, naturally, independently of σ; the dynamics of the scalar field parameters, in particular, of the quantities θ and L_t (Fig. 10) depends on the initial values of the scale ratio parameter

$$R^0 = \frac{\tau_u^0}{\tau_t^0} = \frac{\overline{q^2}^0}{\overline{\rho^2}^0}\frac{\varepsilon_\rho^0}{\varepsilon_u^0} = \frac{6}{5}\frac{L_u^0}{L_\rho^0},$$

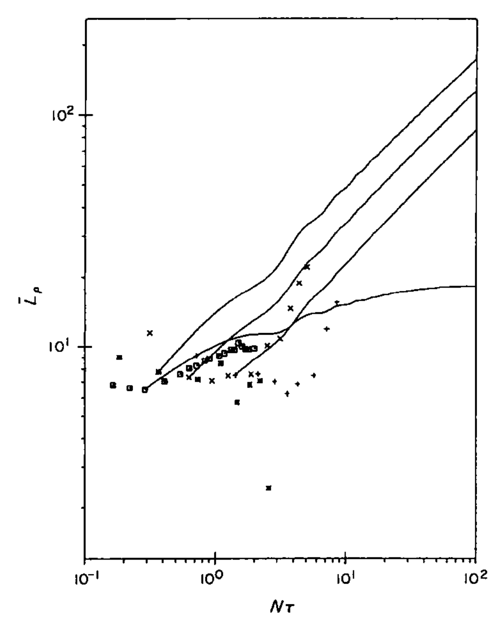


FIG. 10. Evolution of the Taylor macroscale of density fluctuations, $\bar{L}_\rho = L_\rho/M$. For designations see Fig. 1.

which in experiments for air and water differed from each other. In other words, the difference of the exponents in the law of the evolution of the scalar field turbulence parameters in the region of $N\tau < 1$ is due not to the difference in the values of σ, but rather to the initial value of the scale ratio parameter R^0.

As noted above, in the 'collapse' region, a 'trough' in the laws of the evolution of turbulence parameters occurs which is conditioned by the appearance of internal waves. Downstream of this 'trough' the parameters of the field which is the superposition of the turbulence itself and random internal waves, are evolving conditionally self-similarly, i.e. self-similarly for the parameters smoothed with respect to the amplitude of oscillations. However, the rate of self-similar evolution of the corresponding parameters differs substantially for air and water. Thus, from Fig. 3 it follows that the exponent of decay rate averaged over the amplitude of turbulence kinetic energy oscillations n in the power 'law'

$$\overline{q^2} \sim (N\tau)^{-n}$$

is approximately equal to 0.8 for water and to about 1.63 for air. Such a substantial difference in the rate of kinetic energy decay after the collapse can be attributed, on the one hand, to a different contribution of random internal long waves into the superpositional field: the small decay rate of the parameter $\overline{q^2}$ for water indicates a great contribution into this parameter of the energy of random internal gravitational waves favouring the production of large-scale weakly dissipating structures.

On the other hand, a relatively large rate of turbulent kinetic energy decay in air, as compared with water and also with the case of the non-stratified medium, cannot be indeed associated with the presence of internal waves. It seems to be attributed to the relatively low inertia of the scalar field (the evolution of the turbulence Peclet number is presented in Fig. 8). As is known, to a weaker turbulence inertia there corresponds a more rapid decay of it. Therefore, an earlier transition to the final stage is observed in air than in a fluid.

Thus, the influence of the molecular Prandtl number on the evolution of the velocity field parameters shows up through the inertia of the scalar field, i.e. turbulence Peclet number P_λ: the higher the value of this parameter the greater the contribution of the internal waves into the perturbed velocity field.

From Fig. 5 it follows that the molecular Prandtl number exerts an influence also on the degeneration of the mean square of scalar pulsations. Starting from the value of $N\tau$ corresponding to the 'collapse', the rates of decay of the amplitude-averaged values of $\overline{u_2^2}$ and $\overline{\rho^2}$ (or $\overline{t^2}$ for air) turn out to be identical for a given value of σ, so that the ratio of the kinetic to potential energy, being an oscillating function of time, has a constant mean value which is independent either of the global Froude number or of the molecular Prandtl number (see Fig. 6).

It seems that this fact (which was not covered in earlier relevant publications of other authors) has a great importance for correlating the dynamics of moderately strong homogeneous turbulence of stably stratified fluid.

3.3. *Transition of the stably stratified fluid turbulence to the final evolution stage*

As is known from the dynamics of the homogeneous turbulence of a non-stratified fluid, and in compliance with the results of the present work, the parameters R_λ and P_λ decay at a large evolution time (see Figs. 7 and 8), i.e. during its evolution the turbulence of a stably stratified fluid loses its inertia and goes over to the final stage of degeneration or, in conformity with the oceanographic terminology, to the state of 'fossil' turbulence.

Of principal importance for understanding the character of 'fossil' turbulence is the study of the evolution of turbulence in the transition region, i.e. at moderate values of R_λ and P_λ parameters.

To carry out a comprehensive study of the evolution of stably stratified turbulence within this transition range of the parameters R_λ and P_λ, the authors analysed the results of numerical simulation of two experimental realizations which differed by molecular Prandtl numbers and had almost the same global Froude numbers: the experiment of Itsweire *et al.* for $\sigma \simeq 900$, $Fr = 3.7 \times 10^{-2}$ and that of Lienhard and Van Atta for $\sigma \simeq 0.73$, $Fr = 2.4 \times 10^{-2}$. It follows from the data given in Figs. 11 and 12, that the stratified turbulence approaches the final stage at substantially different rates: for liquid the state of moderately strong turbulence persists up to $N\tau \simeq 10^{17}$, while for air the final stage of decay sets in already at $N\tau \simeq 10^4$.

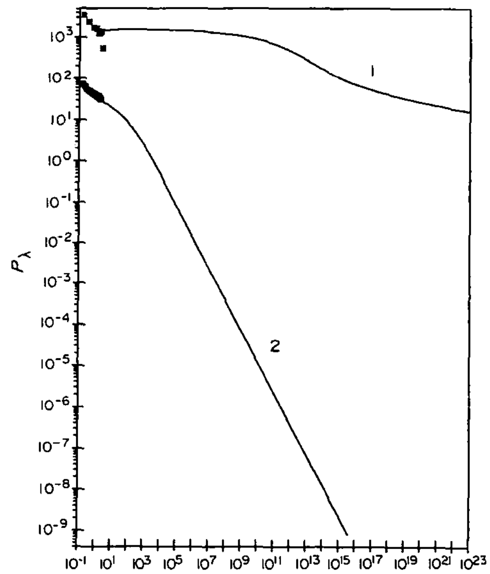


FIG. 12. Evolution of turbulence Peclet number at the transition and final stages. For designations see Fig. 1.

As the inertias of the velocity and scalar fields decrease differently in time, the turbulence energy (Fig. 13) also decays differently in the media considered. At the final stage of degeneration of $\overline{q^2}$ for air, setting in at $N\tau \simeq 10^4$, the degeneration rate exponent in the power law is equal to 5/2 just as for non-stratified medium. For liquid, the degeneration rate exponent of $\overline{q^2}$ at the final stage, which sets in at

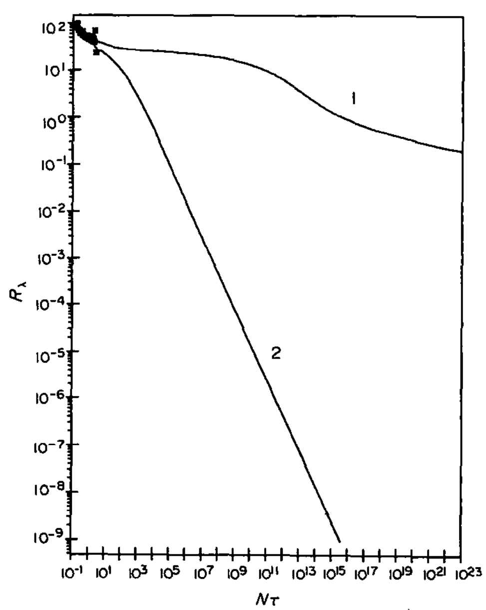


FIG. 11. Evolution of the turbulence Reynolds number at the transition and final stages. Numerical simulation for water (1) and air (2). For designations see Fig. 1.

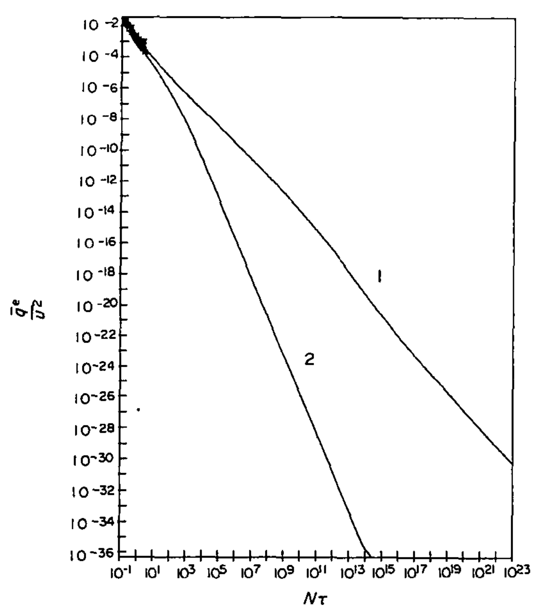


FIG. 13. Evolution of total kinetic energy of turbulence at the transition and final stages. Numerical simulation for water (1) and air (2). For designations see Fig. 1.

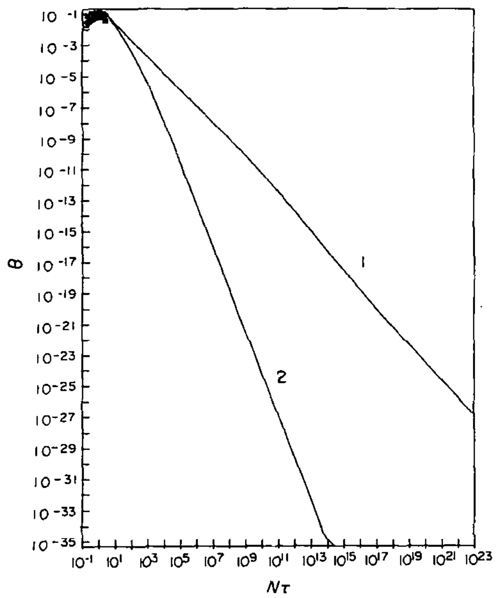


FIG. 14. Evolution of density fluctuations at the transition and final stages. Numerical simulation for water (1) and air (2). For designations see Fig. 1.

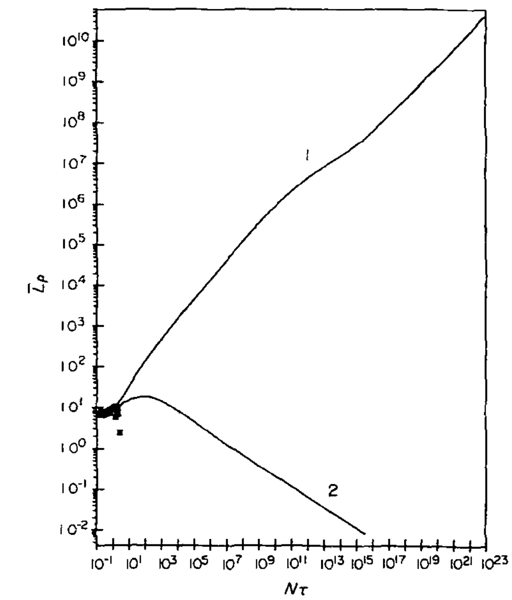


FIG. 16. Evolution of the Taylor macroscale of density fluctuations at the transition and final stages. Numerical simulation for water (1) and air (2). For designations see Fig. 1.

$N\tau \simeq 10^{17}$, has the value twice as small as that for air.

The fluctuations of density (or temperature), shown in Fig. 14, decay in the far region qualitatively in the same fashion as the kinetic energy of turbulence. In the region of small parameters R_λ and P_λ the decay rate exponents for the parameter $\overline{\rho^2}$ for water and $\overline{t^2}$ for air are nearly equal to the corresponding values of the decay rate index of the parameter $\overline{q^2}$.

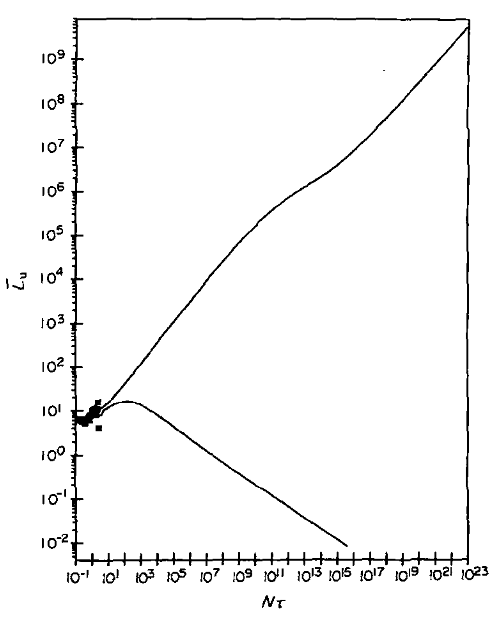


FIG. 15. Evolution of the Taylor macroscale of velocity fluctuations at the transition and final stages. Numerical simulation for water (1) and air (2). For designations see Fig. 1.

The behaviour of the length scales of energy containing vortices, i.e. of the Taylor macroscales L_u and L_ρ, is of fundamental importance for understanding the character of velocity and scalar fields in the far region. As follows from Figs. 15 and 16, the behaviour of the macroscales of length for air and water is different in principle: when for air the parameters L_u and L_t evolve similarly to the isotropic case (or in the case of a homogeneous passive scalar field for the parameter L_t), then for water the indicated parameters increase infinitely. This fact can be presumably explained by a different contribution of internal waves into the far field for the two cases considered: when $N\tau \gg 1$, the effect of the internal waves on the turbulence of air is insignificant, while in liquid the internal waves become dominant. Even though such an assumption has an euristic character, it is confirmed by the analysis of the evolution of other turbulence parameters. Most impressive is the evolution of the ratio of the kinetic energy of vertical fluctuations to the total kinetic energy of the velocity pulsations (Fig. 17). As follows from the numerical results when $N\tau \gg 1$ the vertical velocity fluctuations in air become suppressed earlier than is the case with longitudinal and lateral fluctuations, i.e. the velocity field becomes nearly two-dimensional. For water, the pattern is opposite: at large values of $N\tau$ the vertical velocity fluctuations decay more slowly than the other two components, so that the parameter $\overline{u^2}/\overline{q^2}$ approaches unity, i.e. in the case considered the velocity field asymptotically approaches the 'quasi-one-dimensional' field.

The above-described presumed pattern of the 'fossil' turbulence structure is confirmed by the analysis of the evolution of the ratio between the kinetic energy

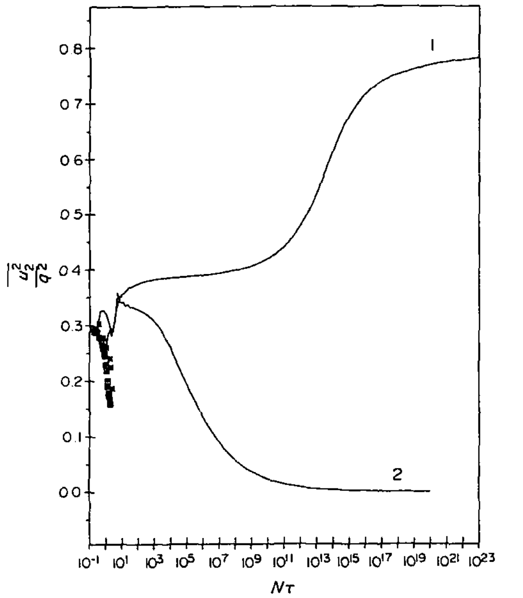


FIG. 17. Evolution of the ratio of the kinetic energy of transverse velocity fluctuations to the total kinetic energy of turbulence at the transition and final stages. Numerical simulation for water (1) and air (2). For designations see Fig. 1.

of vertical velocity disturbances and the potential energy (Fig. 18). From this figure it follows that in the case of turbulence formation in liquid the parameter K/P asymptotically (when $N\tau \to \infty$) tends to zero (or to a certain small value). This testifies to the dominating role of internal waves in the superposition velocity field for $N\tau \gg 1$. In the case of an air at $N\tau \gg 1$ the parameter K/P asymptotically goes over from the region $K/P < 1$, where the contribution of internal waves into the superposition velocity field somewhat exceeds the contribution of the turbulence proper, to the region with the value $K/P = 1$ indicating parity of the energies of vertical turbulent fluctuations and internal waves. However, as the data of Fig. 17 indicate, the contribution of vertical perturbations (both of turbulence and internal waves) into the three-dimensional velocity field is insignificant (turbulence tends to a two-dimensional or horizontal one), i.e. in Fig. 18 the information-bearing curve (in the sense of the analysis of the contribution of internal waves into the relict turbulence) is only that one which corresponds to liquid: it points to the dominating role of internal waves in the fossil turbulence of a stably stratified viscous fluid.

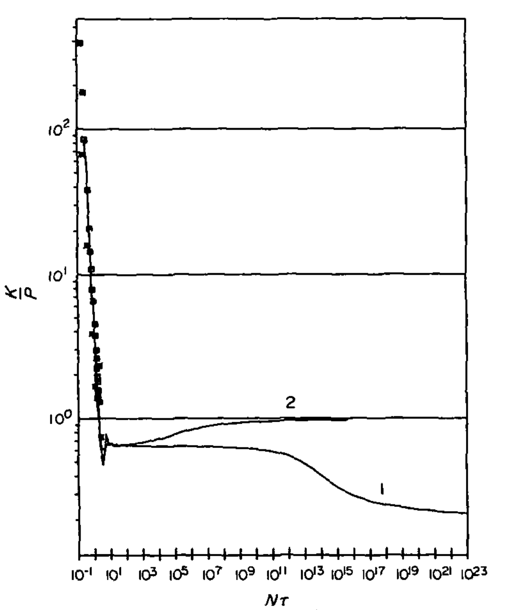


FIG. 18. Evolution of the ratio of the kinetic energy of vertical velocity fluctuations and the potential energy at the transition and final stages. Numerical simulation for water (1) and air (2). For designations see Fig. 1.

Thus, the analysis of the results obtained from numerical modelling of turbulence evolution in stably stratified media shows that, depending on the molecular Prandtl number, the fossil turbulence, i.e. the turbulence of stably stratified media with $R_\lambda \ll 1$ and $P_\lambda \ll 1$, may represent either a quasi-two-dimensional field with the dominating contribution of random two-dimensional disturbances and with a slight 'impurity' of internal waves (for gaseous media), or 'quasi-one-dimensional' field with the dominating contribution of vertical internal waves and insignificant impurity of three-dimensional turbulence (for liquid media).

4. STRUCTURE OF THE TURBULENCE OF STABLY STRATIFIED MEDIA

The above analysis allows one to assume that the perturbed field in stably stratified media is a superposition of the turbulence proper and of the internal gravitational waves, with the contribution of the internal waves into the superpositional field being different depending on the molecular Prandtl number. It should be noted, however, that the aforegoing conclusion about the contribution of internal waves into perturbed velocity field at $N\tau \gg 1$ has a heuristic character, since the above-considered functions in the far region are represented by smooth curves without any signs of wavy motion either in the liquid, or in the gas (see Figs. 17–19). Moreover, the evolution of a transverse mass flux, serving as an indicator of wavy motion of the medium, demonstrates (see Fig. 19) the absence of any signs of scalar field anisotropy at all in the far region.

In view of this, there arises the necessity of carrying out a detailed analysis of the character of change in the process of time evolution of the perturbed field parameters within short intervals of the argument $N\tau$. This might confirm the existence of internal waves and would allow one to elucidate their contribution to the perturbed velocity and density fields.

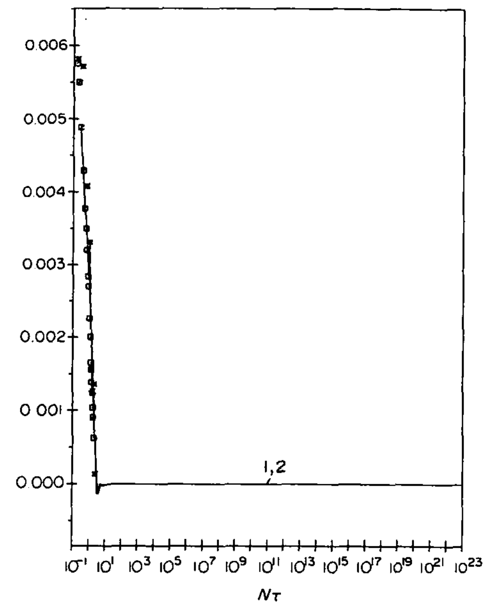


FIG. 19. Evolution of the transverse mass (or heat) flux q at the transition and final stages. For designations see Fig. 1.

In accordance with the data considered in the previous sections, when $N\tau < 1$, the stably stratified medium turbulence parameters evolve qualitatively in the same manner as in the case of independent velocity and scalar fields. When $N\tau \simeq 2.5$–3.0, there occurs a conventional collapse of turbulence which is manifested by a vanishing transverse mass flux and which points to the formation of internal gravitational waves.

Due to the coupling of the velocity and density fields in a stratified medium, the presence of internal waves in the superposition field must show up in the evolution of all the turbulence parameters relating to both velocity and density fields. In fact, as follows from the curves presented in Fig. 20, it is already at the early stage of the evolution of a stably stratified turbulence, including even the region before the collapse, that wavy-like oscillations in the turbulence parameters become observable, as well as the oscillations of the function q about the zero value. The period of such oscillation at the initial stage of evolution is independent of the molecular Prandtl number and is approximately equal to

$$T \simeq 0.56 T_{BV}.$$

A distinctive feature of the development of turbulence in a stably stratified liquid in the initial period is the countergradient vertical mass flux. In fact, as follows from Fig. 21(a), the averaged (over a number of 'periods') value of the parameter q is negative. In this very region of the dimensionless time $N\tau$ the considered parameter for the gas is on average equal to zero.

In the transition region determined by the moderate Reynolds and Peclet numbers and extending over the intervals 10^{10}–10^{20} for liquid and 10^{2}–10^{12} for air (see Fig. 18), the tendency in the development of the parameter q (see Fig. 21) is preserved, i.e. the countergradient transverse mass flux is observed for liquid and, on the average, zero heat flux for air.

At the final stage of evolution determined by small values of the turbulent Reynolds and Peclet numbers

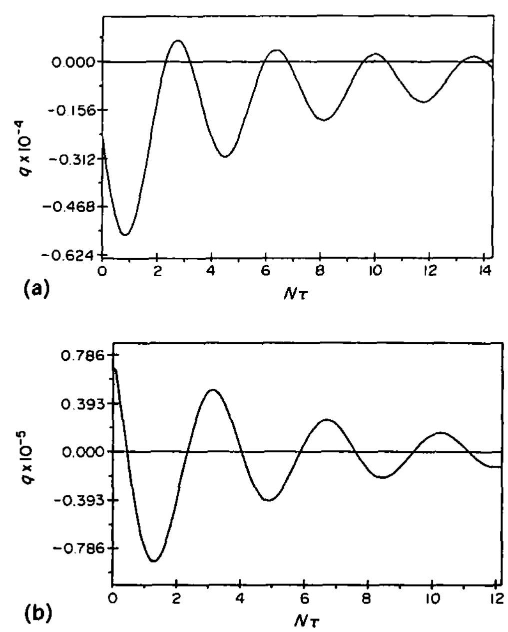


FIG. 20. Evolution of the transverse mass (or heat) flux $q = \overline{u_2 \rho}/UM(-d\bar{\rho}/dx_2)$ at the beginning of the transition stage: (a) water, $(N\tau)_0 = 10.0$; (b) air, $(N\tau)_0 = 10.0$

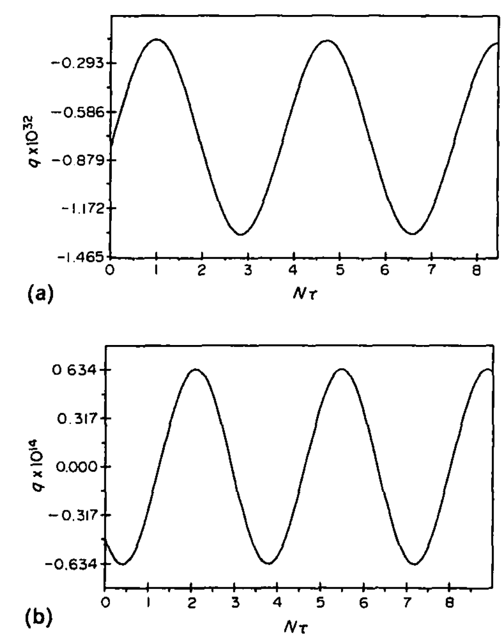
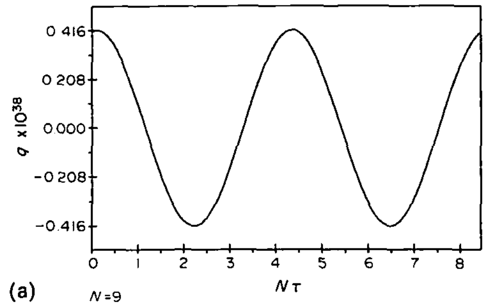


FIG. 21. Evolution of a transverse turbulent mass flux in the transition stage: (a) water, $(N\tau)_0 = 10^{14}$; (b) air, $(N\tau)_0 = 10^{5}$.



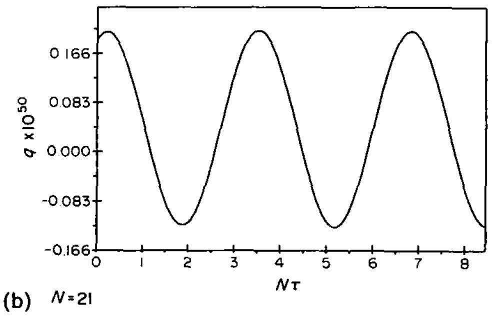


FIG. 22. Evolution of the transverse density (or heat) flux at the final stage: (a) water, $(N\tau)_0 = 10^{22}$; (b) air, $(N\tau)_0 = 10^{14}$.

and extending over the region $N\tau > 10^{20}$ liquid and $N\tau > 10^{12}$ for air, the wavy character of the evolution of the transverse turbulent mass or heat flux is preserved (see Fig. 22). Nevertheless, the qualitative difference of the evolution of this function in the stage considered, in contrast to the previous stage, consists in the absence, on average, of turbulent mass flux for liquid and in the recovery of the downgradient heat flux for air. The period of fluctuations of the function $q(N\tau)$ for water somewhat increased

$$T \simeq 0.7T_{BV}$$

whereas for air it virtually did not change.

Thus, both in the transition region and in the final stage of the evolution of density-stratified turbulence, gravitation plays an important role in the formation of a perturbed velocity field:

- at large molecular Prandtl numbers characteristic for liquid media the internal waves play the dominating role in vertical perturbations of the velocity field; decaying more slowly than turbulent velocity fluctuations (due to the smaller dissipative nature of large scale structures of wave character), the internal waves lead to the formation of a 'quasi-one-dimensional' field with $N\tau \to \infty$ in which the intensity of vertical perturbations due to internal waves exceeds the intensity of longitudinal fluctuations by about an order of magnitude;
- at molecular Prandtl numbers of the order of unity characteristic for gases, the internal waves do not play the dominating role in the superposition velocity field; the buoyancy effect is reduced to the suppression of vertical velocity perturbations (both of internal waves and proper turbulent fluctuations) so that with $N\tau \to \infty$ the velocity field represents a quasi-two-dimensional (horizontal turbulence) with an insignificant 'impurity' of internal waves.

5. CONCLUSION

In the present paper the results are presented of modelling the evolution of homogeneous turbulence in stably stratified media at any distances from an arbitrary origin, for example a grid being a generator of the turbulent isotropic velocity field. The source of the scalar field turbulence is a constant transverse mean density (or temperature) gradient which generates scalar quantity perturbations. The joint correlation of the transverse (in the direction of the averaged density gradient) velocity and density fluctuations or a turbulent transverse mass (heat) flux, which is a source term in the equations for turbulent stresses essentially determines the specific features of the evolution of turbulence of a stably stratified fluid.

5.1. *Specific features of the evolution of stably stratified turbulence in the initial period*

1. The fundamental aspect of the evolution of the considered homogeneous turbulence is a wavy-like alternating variation of the joint correlation $\overline{u_2\rho}$, i.e. the alteration in time of the 'down-gradient' and 'countergradient' mass (or heat) fluxes with the period $T \simeq 3.5N^{-1} \simeq 0.5T_{BV}$ independent of the Froude number. The results of the simulation carried out in the present work show that this feature is typical of flows with any, including small, Brunt–Väisälä frequencies. However, the amplitude of fluctuations of the above-mentioned function decreases with the Froude number $Fr = NM/U$. Thus, when considering the experimental confirmation of this feature, one should bear in mind that at small Froude numbers the amplitude of the oscillations of the parameter $\overline{u_2\rho}$ can be rather small and located within the accuracy range of measurement which makes it impossible to reveal the presence of fluctuations directly from the experiment. This situation seems to be typical for some realizations corresponding to small Froude numbers, of the experiments of Lienhard and Van Atta [5] and also of Yoon and Warhaft [11] in which the changing of the sign of the parameter $\overline{u_2\rho}$ is not evident.

2. The sign of the dimensionless transverse mass flux averaged over a large interval of $N\tau$ depends on the contribution of the internal waves into the perturbed field: with the internal waves playing an insignificant role a down-gradient turbulent transport prevails; in the case of a substantial contribution of the internal waves the countergradient transport prevails (the sign of the turbulent mass flux is identical with that of the mean density gradient).

3. In the region considered, due to the wave-like oscillations of the transverse mass flux, all the ensemble averaged parameters of turbulence of both the velocity and scalar field evolve in a wavy manner with the same period as that of the function $\overline{u_2\rho}$. In this case the value $\overline{u_2\rho} = 0$, repeating with the above-mentioned period, does not at all mean the suppression of the vertical velocity perturbations. Moreover, a substantial contribution of internal waves, as evidenced by wavy oscillations in turbulence parameters, leads to a slower decay of the kinetic energy of velocity fluctuations than in the case of passive scalar. This is quite natural, since the superposition field with internal waves is less dissipative (of larger scale) than a purely turbulent velocity field.

4. The confirmation of the substantial contribution of internal waves into the superpositional field is a jumpwise change in the increasing Taylor macroscales of velocity and scalar fields starting from $N\tau \simeq 2.5$, i.e. from the instant when the transverse mass flux starts to perform alternating oscillations.

5. The differences in the molecular Prandtl numbers typical of the experiments in different media (air and liquid) lead to substantially differing turbulence Peclet numbers. This explains the difference in the rate of evolution of turbulence parameters in various experiments conducted with different media.

6. The amplitude-averaged value of the ratio between the kinetic energy of transverse velocity fluctuations and the potential energy of the density field in a strong turbulence is a universal constant independent either of the Froude number, or of the molecular Prandtl number and is approximately equal to 0.65.

5.2. *Specific features of density-stratified turbulence in transition to the final stage*

1. The transition zone is characterized by a substantial non-self-similarity of the evolution of turbulence parameters.

2. The extension of the transition region depends greatly on the molecular Prandtl number: the larger it is, the higher the value of σ. One fails to relate this fact only to a relatively greater inertia of turbulence in liquid; here the dominating role is played by internal waves, the contribution of which into the superpositional perturbed field in liquids is more substantial than in gases.

3. In the region considered the macroscales of the turbulence of velocity and density fields evolve differently for the two media considered: for air these are the functions with the maximum in the region of moderate values of the parameters R_λ and P_λ, just as in the case of a passive scalar; for a liquid these are infinitely increasing functions with respect to $N\tau$. This fact is an indirect confirmation of the different contribution of internal waves into the superposition perturbed field in the far region for liquids and gases: in a stably stratified gas the contribution of internal waves into the perturbed field is insignificant; at the same time, long internal waves in the perturbed far field of a stably stratified liquid are dominating.

4. The previous statement is supported by a number of factors which, in particular, include the rate of turbulent energy decay in a gas (Fig. 13) the exponent of which in the power law is equal to $(-5/2)$. For liquid it is twice as small thus indicating the dominating role of larger, i.e. less dissipative, structures in a perturbed field formed in a stably stratified liquid.

5. A fundamentally important feature of the evolution of stably stratified turbulence in various media is manifested in the ratio of the kinetic energy of transverse fluctuations to the total kinetic energy of velocity perturbations (see Fig. 17): for liquid this parameter tends to a value close to unity; for air, to zero. This means that the 'fossil' turbulence in liquid is formed in the main by internal waves, while in air by a two-dimensional (horizontal) turbulence.

6. The evidence of the dominating contribution of internal waves into the superposition velocity field in liquid at a high evolution time is the fact that the ratio of the kinetic energy of transverse velocity fluctuations to the potential energy tends to a certain value smaller than unity.

7. The qualitative difference between the perturbed velocity fields in liquid and gas at large evolution times consists in the fact that the 'fossil' (or 'relict') turbulence (i.e. the turbulence of stably stratified media at $R_\lambda \ll 1$ and $P_\lambda \ll 1$) represents either quasi-two-dimensional random velocity perturbations (for gaseous media), or a quasi-one-dimensional velocity field caused by the dominating contribution of vertical internal waves.

5.3. *The structure of turbulence of stably stratified media*

1. The oscillation change in time of the parameters of turbulence is the specific feature of its evolution in a stably stratified medium even at an early stage of its development up to the time when the parameter $\overline{u^2\rho}$ converts into zero for the first time.

2. At the initial stage of turbulence development the sign of the transverse mass flux, averaged over some number of the fluctuation periods, depends on the molecular Prandtl number: countergradient mass transfer prevails in liquid, whereas in gas there is virtually no transverse mass flux.

3. The period of the oscillations of functions in the initial stage of evolution is independent of the molecular Prandtl number and is approximately equal to

$$T \simeq 0.56 T_{BV}.$$

4. In the transition region the character of change in time of turbulence parameters is qualitatively similar to the evolution of turbulence in the initial period.

5. At the final stage of turbulence evolution, the wavy character of the turbulence parameters of the velocity and scalar fields is preserved. However, the

amplitude of the oscillations of these parameters for liquid is much in excess of that for air.

6. For liquid media the period of the perturbations of turbulence parameters at the final stage somewhat increases as compared with the previous, transition, zone.

7. Turbulent mass flux, averaged over a number of periods of oscillations is virtually absent for liquid media, whereas for gas it adopts a conventional sign inherent in down-gradient transfer.

5.4. *Concluding remark on the role of stratification in the formation of a perturbed field at high turbulence evolution times*

The role of gravitation in the homogeneous turbulence of a stably stratified media manifests itself via the 'participation' of internal waves in the superposition perturbed velocity field which is reduced to the formation, at large evolution times, of a 'quasi-one-dimensional' perturbed field in a liquid with predominance of wave-type perturbations and of a quasi-two-dimensional (horizontal) field in a gas with predominance of random turbulence perturbations.

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